Monotone Conformance Checking for Partially Matching Designed and Observed Processes

Artem Polyvyanyy and Anna Kalenkova
1st International Conference on Process Mining, June 24-26, 2019, Aachen, Germany
Conformance Checking

**Conformance checking** relates events in the event log to activities in the process model and compares both. The goal is to find commonalities and discrepancies between the modeled behavior and the observed behavior [1].

<table>
<thead>
<tr>
<th>Event log 1</th>
<th>Process model 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trace</strong></td>
<td><strong>Event log</strong></td>
</tr>
<tr>
<td>ABDEI</td>
<td>1,207</td>
</tr>
<tr>
<td>ACDGHFI</td>
<td>145</td>
</tr>
<tr>
<td>ACGDHFI</td>
<td>56</td>
</tr>
<tr>
<td>ACDHFI</td>
<td>28</td>
</tr>
<tr>
<td>ACHDFI</td>
<td>23</td>
</tr>
</tbody>
</table>

**Precision:** Process model *should not* allow for behavior unrelated to what was seen in the event log.

**Fitness:** Process model *should* allow for the behavior seen in the event log.

---

Conformance Checking (II)

Single trace model

Event log 1

<table>
<thead>
<tr>
<th>Trace</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABDEI</td>
<td>1,207</td>
</tr>
<tr>
<td>ACDGHFI</td>
<td>145</td>
</tr>
<tr>
<td>ACGDHFI</td>
<td>56</td>
</tr>
<tr>
<td>ACDHFI</td>
<td>28</td>
</tr>
<tr>
<td>ACHDFI</td>
<td>23</td>
</tr>
</tbody>
</table>

+ fitness
- precision

Flower model

+ fitness
- precision

Process model 2

Process model 3

+ fitness
- precision

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Precision and Recall in Information Retrieval

Query

Document collection

Information Retrieval engine

Relevant documents

Retrieved documents

Precision:

\[ \frac{3}{4} \]

Recall:

\[ \frac{3}{5} \]
Precision and Recall in Conformance Checking

Precision:
\[
\frac{|L \cap M|}{|M|} = \frac{5}{6}
\]

Recall (Fitness):
\[
\frac{|L \cap M|}{|L|} = \frac{5}{5}
\]

\[M = L \cup \{\text{ACGHDFI}\}\]
Two Problems of Precision and Recall

Problem 1: Infinite number of traces

Event log 1
Trace
ABDEI
ACDGHFI
ACGDFHI
ACHDFI

Process model 2

Precision:
\[
\frac{|L_1 \cap M_2|}{|M_2|} = \frac{5}{\infty} = 0
\]

Recall:
\[
\frac{|L_1 \cap M_2|}{|L_1|} = \frac{5}{5} = 1
\]

Problem 2: Partially matching traces

Event log 2
\{ABBCE\}

Process model 4

Precision:
\[
\frac{|L_2 \cap M_4|}{|M_4|} = 0
\]

Recall:
\[
\frac{|L_2 \cap M_4|}{|L_2|} = 0
\]
Problem 1: Infinite Number of Traces

**Solution:** Estimate the number of traces using the notion of topological entropy [12]:

\[ \text{ent}(B) := \lim_{n \to \infty} \sup \frac{\log |C_n(B)|}{n} . \]

- $B$ is an ergodic deterministic finite automaton;
- $C_n(B)$ is the set of all traces of $B$ of length $n$;
- ent($B$) is equal to a Perron-Frobenius eigenvalue of the adjacency matrix of $B$;
- ent•($B$) := ent($\hat{B}$), where $\hat{B}$ is a short-circuit version of $B$.

**Precision [8]:** \[ \frac{\text{ent}•(L \cap M)}{\text{ent}•(M)} \]

**Recall [8]:** \[ \frac{\text{ent}•(L \cap M)}{\text{ent}•(L)} \]

---

Entropy-Based Precision and Recall Examples

Magic numbers ?!

<table>
<thead>
<tr>
<th>Model</th>
<th>Log</th>
<th>prec</th>
<th>recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>$L_1$</td>
<td>0.874</td>
<td>1.000</td>
</tr>
<tr>
<td>$M_1$</td>
<td>$L_2$</td>
<td>1.000</td>
<td>0.745</td>
</tr>
<tr>
<td>$M_1$</td>
<td>$L_3$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$M_2$</td>
<td>$L_1$</td>
<td>0.754</td>
<td>1.000</td>
</tr>
<tr>
<td>$M_2$</td>
<td>$L_2$</td>
<td>0.863</td>
<td>0.745</td>
</tr>
<tr>
<td>$M_2$</td>
<td>$L_3$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Properties of Conformance Checking Measures

<table>
<thead>
<tr>
<th>Metric</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple behavioral appropriateness</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Advanced behavioral appropriateness</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>One-align ETC</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Negative event precision</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>PCC precision</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anti-alignment precision</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Entropy-based precision: ✓ ✓ ✓ ✓ ✓ ✓

Precision properties [10]:

A1: Precision is deterministic;
A2:

\[
\text{prec}(L_1, M_2) \leq \text{prec}(L_1, M_1)
\]

A3: Precision between an event log and the flower model (over the same alphabet) is the lowest;
A4: Precisions of an event log on two language equivalent models should be equal;
A5:

\[
\text{prec}(L_1, M_1) \leq \text{prec}(L_2, M_1).
\]

A2' [8]:

\[
\frac{\text{ent} \bullet (L_1 \cap M_2)}{\text{ent} \bullet (M_2)} < \frac{\text{ent} \bullet (L_1 \cap M_1)}{\text{ent} \bullet (M_1)}
\]

A5' [8]:

\[
\frac{\text{ent} \bullet (L_1 \cap M_1)}{\text{ent} \bullet (M_1)} < \frac{\text{ent} \bullet (L_2 \cap M_1)}{\text{ent} \bullet (M_1)}
\]


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Entropy-Based Precision and Recall Revisited

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<td>0.000</td>
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<td>$L_3$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Problem 2: Partially Matching Traces (I)

\[ \text{Precision: } \frac{\text{ent}(L \cap M)}{\text{ent}(M)} \]

\[ \text{Recall: } \frac{\text{ent}(L \cap M)}{\text{ent}(L)} \]
**Problem 2: Partially Matching Traces (II)**

**M**

```
A -> B -> C <- D -> E
C <- B
```

**L**

```
A -> B -> B -> C -> E
B <- C
```

**Precision:** \[
\frac{\text{ent}(L \cap M)}{\text{ent}(M)}
\]

**Recall:** \[
\frac{\text{ent}(L \cap M)}{\text{ent}(L)}
\]

“Dilute” model and log traces to identify commonalities:

- Model traces: ABCD, ACBE
- Log traces: ABCE, ACBE

- \(L\): ABCE
- \(M\): ABCD

- \(L \cap M\): ABCE
Problem 2: Partially Matching Traces (III)

\[ \text{Precision: } \frac{\text{ent} \cdot (L \cap M)}{\text{ent} \cdot (M)} \]

\[ \text{Recall: } \frac{\text{ent} \cdot (L \cap M)}{\text{ent} \cdot (L)} \]

"Dilute" model and log traces to identify commonalities:

\[ L \]

\[ L' \]

\[ M \]

\[ M' \]
Problem 2: Partially Matching Traces (IV)

Precision_\tau: \frac{\text{ent} \cdot (L' \cap M')}{\text{ent} \cdot (M')}

Recall_\tau: \frac{\text{ent} \cdot (L' \cap M')}{\text{ent} \cdot (L')}

“Dilute” model and log traces to identify commonalities:
Entropy-Based Precision and Recall Revisited

<table>
<thead>
<tr>
<th>Model</th>
<th>Log</th>
<th>prec</th>
<th>recall</th>
<th>prec_τ</th>
<th>recall_τ</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_1)</td>
<td>(L_1)</td>
<td>0.874</td>
<td>1.000</td>
<td>0.873</td>
<td>1.000</td>
</tr>
<tr>
<td>(M_1)</td>
<td>(L_2)</td>
<td>1.000</td>
<td>0.745</td>
<td>1.000</td>
<td>0.960</td>
</tr>
<tr>
<td>(M_1)</td>
<td>(L_3)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.873</td>
<td>0.811</td>
</tr>
<tr>
<td>(M_2)</td>
<td>(L_1)</td>
<td>0.754</td>
<td>1.000</td>
<td>0.615</td>
<td>1.000</td>
</tr>
<tr>
<td>(M_2)</td>
<td>(L_2)</td>
<td>0.863</td>
<td>0.745</td>
<td>0.704</td>
<td>0.960</td>
</tr>
<tr>
<td>(M_2)</td>
<td>(L_3)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.733</td>
<td>0.966</td>
</tr>
</tbody>
</table>
A Selection of New Properties

**T3:**
\[
\frac{\text{ent}\cdot(L_1 \cap M_1)}{\text{ent}\cdot(M_1)} \leq \frac{\text{ent}\cdot(L_2 \cap M_1)}{\text{ent}\cdot(M_1)}
\]

**T5:**
\[
\frac{\text{ent}\cdot(L_1 \cap M_1)}{\text{ent}\cdot(M_1)} < \frac{\text{ent}\cdot(L_2 \cap M_1)}{\text{ent}\cdot(M_1)}
\]

* For each k, there are at least as many words of length k in \(L_2 \cap M_1\) as in \(L_1 \cap M_1\), and sometimes more.

**T6:**
\[
L' = M' \quad \text{**A possible outcome**}
\]
\[
\text{ent}\cdot(L') \leq \text{ent}\cdot(M')
\]

Consequently, **A2** and **A5** hold.

**T7:**
\[
\exists \alpha \in M: \alpha \notin L' \quad \text{and} \quad \forall \beta \in L: \beta \in M'
\]
\[
L' \subset M' \quad \text{and} \quad \text{ent}\cdot(L') < \text{ent}\cdot(M')
\]

Consequently, **A2'** and **A5'** hold.
### Entropy-Based Precision and Recall Revisited

#### Model Comparison Table

<table>
<thead>
<tr>
<th>Model</th>
<th>Log</th>
<th>prec</th>
<th>recall</th>
<th>prec_{\tau}</th>
<th>recall_{\tau}</th>
</tr>
</thead>
<tbody>
<tr>
<td>M_1</td>
<td>L_1</td>
<td>0.874</td>
<td>1.000</td>
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<tr>
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<td>0.615</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.733</td>
<td>0.966</td>
</tr>
</tbody>
</table>

#### Diagram

- **L_1**
- **L_2**
- **M_1**
- **M_2**
- **L_3**
- **ε**
- **AC**
- **BAD**
- **BAB**
- **BA(DA)^+**

- **T3&T7**
- **T5**

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Experiment Using Synthetic Event Data [14,15]

Event log 1

Trace
ABDEI
ACDGHFI
ACGDFHI
ACHDFI

(a) Original model
(b) Single trace model
(c) Separate traces model

(d) Flower model
(e) G and H in parallel model
(f) G and H in loops model

(g) D in a loop model
(h) All parallel model
(i) Round robin model
### Precision for Synthetic Event Data

<table>
<thead>
<tr>
<th>Model</th>
<th>SD</th>
<th>ETC(_a)</th>
<th>NE</th>
<th>PCC</th>
<th>AA</th>
<th>MAP(^1)</th>
<th>MAP(^2-γ)</th>
<th>EB</th>
<th>EB(^τ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>1</td>
<td>7</td>
<td>7</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Single trace</td>
<td>8</td>
<td>8</td>
<td>6</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Separated traces</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Flower</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>H and G in parallel</td>
<td>6</td>
<td>3</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>H and G in loops</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>D in a loop</td>
<td>1</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>All parallel</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Round robin</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- Set Difference (SD);
- Negative Events (NE);
- Alignment-based ETC precision (ETC\(_a\));
- Projected conformance checking (PCC);
- Anti-alignment precision (AA);
- k-order Markovian abstraction (MAP\(^k\));
- Entropy-based precision (EB);
- EB with partial matching (EB\(^τ\)).

**Process model 2**

\[ EB = 0.568 \]
\[ EB^τ = 0.933 \]

**Process model 3**

\[ EB = 0.758 \]
\[ EB^τ = 0.970 \]
Analyzed 5 real-life event logs;
Filtered out infrequent events that appear less than in 80% of traces using Filter Log using Simple Heuristics ProM plugin;
From 596 to 11,636 traces;
From 1,403 to 164,144 events;
The Inductive miner was used to discover models from event logs;
During determinization, automata sizes increased up to ten times;
The experiment was conducted using the Intel Core™ i7-3970X CPU@3.50 GHz, 64 GB RAM;
Tool is publicly available at: https://github.com/akalenkova/eigen-measure.

<table>
<thead>
<tr>
<th>Event log</th>
<th>Automaton</th>
<th># States / # Transitions</th>
<th>Determin. time</th>
<th>Entropy calc. time</th>
<th>Precision / Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPIC’12</td>
<td>$L'$, $M' \cap L'$</td>
<td>90,557 / 446,847, 90,557 / 446,847</td>
<td>141.455, -</td>
<td>4,641.421, 4,733.990</td>
<td>0.709 / 1.000</td>
</tr>
<tr>
<td></td>
<td>$M'$</td>
<td>3 / 33</td>
<td>0.001</td>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td>BPIC’13 closed</td>
<td>$L'$, $M' \cap L'$</td>
<td>216 / 629, 216 / 629</td>
<td>0.171, -</td>
<td>0.235, 0.661</td>
<td>0.960 / 1.000</td>
</tr>
<tr>
<td></td>
<td>$M'$</td>
<td>1 / 3</td>
<td>0.001</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td>BPIC’13 incidents</td>
<td>$L'$, $M' \cap L'$</td>
<td>24,336 / 72,994, 24,336 / 72,994</td>
<td>1,909.794, -</td>
<td>2.187, 1.552</td>
<td>0.995 / 1.000</td>
</tr>
<tr>
<td></td>
<td>$M'$</td>
<td>1 / 3</td>
<td>0.001</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>BPIC’13 open</td>
<td>$L'$, $M' \cap L'$</td>
<td>17 / 31, 17 / 31</td>
<td>0.012, -</td>
<td>0.123, 0.003</td>
<td>0.980 / 1.000</td>
</tr>
<tr>
<td></td>
<td>$M'$</td>
<td>1 / 2</td>
<td>0.000</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>BFS’13</td>
<td>$L'$, $M' \cap L'$</td>
<td>22,359 / 200,254, 7,542 / 45,163, 514 / 3,340</td>
<td>37.947, 1.625</td>
<td>2.427, 0.153</td>
<td>0.939 / 0.825</td>
</tr>
</tbody>
</table>
Contributions, Limitations, and Future Work

Contributions:
- Extension of the entropy-based precision and recall measures to support partially matching traces;
- Properties for conformance measures that address partially matching traces;
- An evaluation that demonstrates the feasibility of using the new measures in industrial settings.

Limitations:
- Efficiency due to the automata determinization step;
- No support of process models that describe non-finite spaces of reachable states;
- Frequencies of traces are ignored.

Future/current work:
- Efficient computation of the entropy of an event log;
- Entropy-based precision and recall for unbounded-state-systems;
- Controlled “dilution” of traces, i.e., precision and recall over traces that differ in up to k events.